Topic Study Group 2

Mathematics Education at Tertiary Level

G. Gueudet[1] and I. Biza[2]

ABSTRACT In this report we summarize the activities and the studies presented at the TSG-2: Mathematics education at tertiary level of the 14th International Congress on Mathematics Education (ICME-14) that took place online and in Shanghai, China on July 11 to 18, 2021. The activities of the group spanned across four themes: mathematics teaching; students' practices and experiences in mathematics; transitions to, across and from studies of mathematics at tertiary level; and, mathematics for other disciplines. New themes and emerging theoretical directions are amongst the suggestions of the group.

Keywords: Mathematics for other disciplines; Secondary-tertiary transition; Students' practices and experiences; Teaching at tertiary level.

1. Introducing TSG-2 at ICME-14

1.1. Scientific scope of TSG-2

Topic Study Group 2 (TSG-2) at ICME-14 aimed to share and discuss the recent results of research and practice on learning and teaching mathematics at tertiary level, and to identify perspectives for future research. The works in the group drew on findings discussed at ICME-13 related to themes such as: mathematical practices; teaching, professional and curriculum development; connections to engineering; transition to university; preservice teachers; student thinking; and, research related to specific courses such as calculus, differential equations, and linear algebra — see Topical Survey on Research on Teaching and Learning Mathematics at the Tertiary Level (Biza et al., 2016).

The scope of TSG-2 was described in the call for papers as follows:

The questions studied can concern "traditional" courses, such as "chalk and talk" lectures in relations to teachers' practices and students' difficulties or achievements. They can also relate to "innovative approaches", such as design, implementation and evaluation of experimental courses. The contributions can address particular mathematical domains or mathematical practices. They can also concern teaching and learning practices such as assessment, use of technologies or resources; or university teachers' professional knowledge and professional development. Also, we expect

[1] University Paris-Saclay, UR Etudes sur les Sciences et les Techniques, Orsay, 91400, France, E-mail: ghislaine.gueudet@universite-paris-saclay.fr

[2] School of Education and Lifelong Learning, University of East Anglia, Norwich, NR47TJ, United Kingdom, E-mail: i.biza@uea.ac.uk

submitted proposals to address the variety of tertiary programs that include mathematics, such as pure mathematics, engineering, teacher education, etc. Contributions can engage explicitly with theory (e.g., cognitive, socio-cultural, institutional, discursive, etc.) and certain methodological approach or can share a systematic reflection on teaching and learning practices."

More precisely, TSG-2 initially proposed the following five themes:

- Mathematics Teaching at the Tertiary Level;
- Students' Practices and Experiences in Mathematics at the Tertiary Level;
- Mathematical Topics Teaching and Learning at Tertiary level;
- Transitions to, across and from Studies of Mathematics at Tertiary Level;
- Mathematics for Other Disciplines at the Tertiary Level.

All these themes were covered by the submissions we received. Nevertheless, the "mathematical topics" (e.g., Linear Algebra, Calculus, Arithmetics etc.) theme was discussed in studies which could be also categorized in one of the other themes. As a result, the Sections 2 to 5 below address the four other themes with attention to the specific mathematical topics under consideration. Finally, Section 6 addresses emerging perspectives for further research that were discussed in the group.

1.2. TSG-2 at ICME-14: Organization and participants

The work in TSG-2 at ICME-14 was prepared by a team including the authors of this report, as chair and co-chair of the group; Rongrong Cao (Qingdao University, China); Victor Giraldo (Universidade Federal do Rio de Janeiro, Brazil); and, Azimeh Khakbaz (Bu-Ali Sina University, Iran). The IPC Liaison person was Frode Rønning (Norwegian University of Science and Technology, Norway). During the hybrid conference, TSG-2 welcomed 1 invited talk (30 minutes); 8 long presentations (15 minutes); 16 short presentations (10 minutes) and 3 posters (brief 'teaser' presentation before the poster sessions). The authors came from more than 20 countries, representing all the different parts of the world (Brazil, Canada, Chile, China, Colombia, Finland, France, Germany, Hong Kong SAR of China, India, Iran, Israel, Netherland, Philippines, Russia, Spain, South Africa, Sweden, Thailand, Uganda, UK, USA, etc.). Tab. 1 (on the next page) lists the titles and the authors of the papers and posters presented.

2. Mathematics Teaching at the Tertiary Level

The communications in TSG-2 highlighted the ever-growing interest in mathematics teaching at the tertiary level. Different complementary aspects were considered by the participants of TSG-2. Watkins et al.[22] studied university teachers' mathematical knowledge for teaching algebra. Analyzing videos of lessons, they evidenced that for instructors with high MKT for teaching algebra, much more segments without errors or imprecisions can be observed. Chen and Niu[26] studied the impact on class size, in the context of an evolution in Chinese universities to reduce this size. While it did not directly impact students' achievement, the class size influenced teaching practices.

Tab. 1. List of papers and posters presented at TSG-2

Paper and author(s)	
[1]	Errors of engineering students on the vector subspace concept. ***Andrea Cárcamo*** *and Claudio Fuentealba* (Chile).
[2]	Transition between paradigms in the university: The role played by the theoretical framework. ***Ignasi Florensa*** *and* ***Marianna Bosch*** (Spain).
[3]	Gendered patterns in university students' use of learning strategies for mathematics. ***Lara Gildehaus*** *and Michael Liebendörfer* (Germany).
[4]	First year university students' goals and strategies. ***Robin Göller*** (Germany).
[5]	Comparing two self-assessment models in a mathematics course — an exploratory study. ***Jokke Häsä****, Johanna Rämö, and Juulia Lahdenperä* (Finland).
[6]	Geometry for student teachers — capstone course in mathematics with a multitude of links to school mathematics. *Max Hoffmann and* ***Rolf Biehler*** (Germany).
[7]	Engineering students' approach to studying mathematics and its influence on their achievement. ***Helena Johansson****, Magnus Oskarsson, and Hugo von Zeipel* (Sweden).
[8]	The quality of mathematics teacher education at tertiary level in Uganda: is it relevant for 21st Century Mathematics Teachers? ***Marjorie Sarah Kabuye Batiibwe*** (Uganda).
[9]	How university students perceive the importance of resources to study calculus and linear algebra. ***Zeger-Jan Kock****, Birgit Pepin* (The Netherlands)*, and Domenico Brunetto* (Italy).
[10]	Success of mathematics training and talent search programme in India. ***Ajit Kumar*** *and S. Kumaresan* (India).
[11]	Conceptualizing agency and autonomy in tertiary mathematics. ***Mariana Levin****, John P. Smith III, Shiv S. Karunakaran, Valentin A.B. Küchle, and Sarah Castle* (USA).
[12]	The relational nature of supports for high priority mathematics students. ***Behailu Mammo*** *and Signe E. Kastberg* (USA).
[13]	BullsEyes and circles: Alternative scoring practices in collegiate mathematics courses. M*ichelle Morgan and* ***Jeffrey J. King*** (USA).
[14]	From student scribbles to institutional script: Towards a commognitive research and reform programme for university mathematics education. ***Elena Nardi*** (UK)*, Irene Biza* (UK)*, Bruna Moustapha-Corrêa* (Brazil)*, Evi Papadaki* (UK)*, and Athina Thoma* (UK).
[15]	The Secondary-Tertiary transition: An international perspective on where we are and how to move forward. ***Alon Pinto****, Hadas Levi Gamlieli, and Boris Koichu* (Israel).
[16]	An innovative hands-on activity to facilitate the learning of group of symmetries in abstract algebra. ***Tika Ram Pokhrel*** *and Parames Laosinchai* (Thailand).
[17]	The double discontinuity in teacher education — How to face it? ***Cydara Cavedon Ripoll*** *and Luisa Rodríguez Doering* (Brazil).
[18]	Instructors, Mentors, and Students: A Cross-comparison of perceptions of student-centered instruction. ***Kimberly Cervello Rogers****, Sean P. Yee, Jessica Deshler, and Robert Petrulis* (USA).
[19]	From a "strict and scary" class to the "active and favorite" subject: A long-lasting change in the teaching of mathematics at a first-year military school in Chile. ***Antonio Salinas Layana****, Sergio Celis, and Farzaneh Saadati* (Chile).
[20]	An approach to transition of mathematics of secondary to tertiary level mathematics. ***Gloria Inés Neira Sanabria*** (Colombia).
[21]	Mentoring of mid-career and early-career faculty. ***James Sandefur****, Michael Raney, Erblin Mehmetaj, and David Ebenbach* (USA).
[22]	Investigating mathematical knowledge for teaching and quality of instruction in US community colleges. ***Laura Watkins****, Irene Duranczyk, Vilma Mesa, and April Ström* (USA).
[23]	Student reasoning about eigenequations in mathematics and quantum mechanics. ***Megan Wawro****, John Thompson, and Kevin Watson* (USA).
[24]	Characteristics of collective mathematical activity associated with states of student engagement. ***Derek A. Williams****, Jonathan López Torres, and Emmanuel Barton Odro* (USA)
[25]	Flipping a general education mathematics course. ***Fei Xue*** *and Robert Nanna* (USA).
[26]	Study of the influence of class size on the teaching effect of college mathematics. ***Chaodong Chen*** *and Dunbiao Niu* (China). (Poster)
[27]	The relationship between conceptual and procedural knowledge. ***Janine Hechter*** (South Africa). (Poster)
[28]	Meaning of good mathematics teaching from the university students' point of view. ***Seyed Hadi Afzali Borujeni*** *and Azimehsadat Khakbaz* (Iran). (Poster)

Nevertheless, some teachers did not adapt their practices for small classes (50–60 students).

Other studies focused on the development of teacher practices, and the support needed, in particular towards more student-centered approaches. Mammo and Kastberg[12] observed a teacher improving his practice especially for underperforming students, in a setting where peer-tutors were helping these students. The teacher developed their awareness of factors that favor or hinder the efficiency of peer-tutoring. Salinas Layana et al.[19] studied the development of teaching practices on long term, towards more student-centered practices. Through interviews with two teachers, they evidenced that the support of the institution and the agency of teachers were both crucial for this development. Indeed, promoting student-centered practices requires specific teacher support and education. Sandefur et al.[21] investigated the impact of an intensive mentoring experience. Two teachers were involved in a course using a flipped approach and more active learning strategies, new to both of them. Teachers were supported by experienced mentors, and used videos and other material prepared by their mentors. The authors observed evolutions of the two teachers' practices, and of their awareness of students' learning processes. Rogers et al.[18] analyzed the teaching practices of novice collegiate mathematics instructors, also supported by mentors for implementing student-centered techniques. The analyses evidenced discrepancies between the declarations of the novice instructors (who consider that they actually used students-centered techniques), and those of the students and of the mentors. Even with the support of mentors, implementing students-centered techniques remains challenging.

Some studies also presented successful interventions. Kumar and Kumaresan[10] presented a program called: "Mathematics Training and Talent Search (MTTS)", and emphasized some of its aspects, such as personal attention and collective work. This four-week summer school for university students in India has convinced many students to go further with their mathematical studies. Pokhrel and Laosinchai[16] presented an innovative hands-on activity for the learning of group of symmetries in undergraduate level. Working on this activity, students developed an inquiry stance and explore groups of symmetries. Xue and Nanna[25] presented a flipped course about modeling with elementary functions. They evidenced that students learned better with this course than with a traditional lecture-based course, and that the classes were more active and dynamic. Some of the interventions concerned the assessment practices. Häsä et al.[5] study the impact of students' self-assessment on their learning practices. Comparing two models of self-assessment, they show that the assessment of their own skills seems to promote deep learning, more than the assessment of coursework. Morgan and King[13] studied alternative scoring practices and how they impact the students' learning experience. They evidenced the importance of feedback, allowing the students to improve their scores; and the positive effect of non-numerical scores.

3. Students' Practices and Experiences in Mathematics at the Tertiary Level

Students' practices and perceptions at tertiary level were also an important theme in TSG-2. Several theoretical evolutions have been proposed, and new themes emerged.

Williams et al.[24] investigated students' engagement during collective mathematical activity. They proposed a specific theoretical construct about students' engagement in the context of collective activity, linked with the interpretive framework introduced by Cobb and Yackel (1996). They observed that participating in argumentation is not sufficient for high engagement, while participation leading to collective progress and further understanding corresponded to higher engagement. The theoretical construct they proposed evidences that sociomathematical norms influence associations between engagement and mathematical activity. Levin et al.[11] proposed a conceptualization of students' agency and autonomous actions. Through interviews with students, they observed that agency and autonomy should not be described as qualities that participants either had or did not have. Agency and autonomy depend on the context, and exist on a continuum. Another theoretical and methodological construct was proposed by Göller[4] to investigate students' goals and strategies. Drawing on self-regulated learning theory, Göller introduced three overarching categories of strategies and associated goals. Learning strategies aimed to understand and remember new content, problem-solving strategies aimed to solve mathematical problems; nevertheless, students also used coping strategies to deal with institutional requirements. An empirical study confirmed the relevance of this theoretical construct. Gildehaus and Liebendörfer[3] also investigated students' strategies, searching for gendered patterns in the choice of strategies. They observed that, across different courses, female students reported a higher use of organization, time investment and peer learning strategies. This confirmed the view of female students being diligent and social, and can explain gender differences in the learning of university mathematics.

Other studies considered students' perceptions. Khakbaz and Afzali Borujeni[28] investigated what "good mathematics teaching" means for students. They observed different meaning, linked with the students' specializations. For example, for engineering students a good teaching of mathematics emphasizes on applications. Kock et al.[9] study concerned students' perception of the usefulness of different kinds of resources to study mathematics. Analyzing students' answers to a survey, they observed three different groups of students, those who see more importance (a) to lecturer explanations; (b) to the textbook; and, (c) to other curriculum resources (e.g., worked examples, materials prepared by the teacher). Nevertheless, these groups were depended on the courses as the resources proposed by the teachers and the institution vary across courses, and can influence students' practices.

4. School and Tertiary Mathematics Education, Transitions

The secondary-tertiary transition was, not surprisingly, one of the issues addressed in TSG-2. Pinto et al.[15] conducted an international survey about this transition, collecting the views of 310 university mathematics teachers in 30 countries. Exploring the discourses of the respondents, they noted that the concerns about the secondary-tertiary transition were aggravated during the last decades; many universities have organized concrete measures to face the difficulties, but communication between university teachers, secondary school teachers and mathematics education researchers still seems to be lacking. Several other papers addressed issues pertaining to teacher education at university, in different countries: Uganda (Kabuye Batiibwe[8]), Colombia (Sanabria[20]), Brazil (Ripoll and Doering[17]) and Germany (Hoffmann and Biehler[6]). Future teachers experience indeed "double discontinuity" as identified by Klein (1908): from secondary school mathematics to university mathematics, and then back to secondary school mathematics. The content of the teacher education programs has to acknowledge this double discontinuity, for example, by connecting university and school mathematics (see Hoffmann and Biehler[6], for an example in geometry).

5. Mathematics for Other Disciplines at the Tertiary Level

Research about mathematics and non-mathematics disciplines has developed internationally in the last five years; communications within TSG-2 at ICME-14 reflect this development. Wawro et al.[23] investigate students' meanings for mathematics (namely eigentheory) when solving quantum physics tasks. Using the theoretical framework of Knowledge-in-Pieces (diSessa 1993), they analyzed students' discourses and evidenced in some cases synergies between the mathematical and physical meaning of the same concept, but also incompatible interpretations. Other studies related to this theme concern mathematics in engineering education. In this context, the combination of theoretical and procedural aspects is a complex issue. Hechter[27] evidenced that conceptual understanding and procedural fluency are intertwined and should not be separated in the mathematics courses for future engineers. Cárcamo and Fuentealba[1] studied the difficulties encountered by engineering students working on linear algebra tasks, and observed difficulties related to proof and definitions. Johansson et al.[7] studied the learning practices of engineering students in a differential calculus course and their consequences in terms of students' achievement. While regular personal work across the semester had positive effects on exam results, some students worked with mathematics in the preparation for the exams only.

6. New Themes and Emerging Theoretical Directions

Studies in TSG-2 evidenced new themes and emerging directions for research. We note that Inquiry-Oriented practices in mathematics at tertiary level seem to be now well developed in the USA. Which kind of teacher support (including teacher education programs) can promote these new practices? This question is not only considered in the USA, for example, *Study and Research Paths* (SRPs, Bosch, 2018) are Inquiry-Based courses that have been increasingly developing in Europe and South America.

SRPs are also linked to emerging theoretical and methodological development we identified at TSG-2's works. The studies about SRPs are grounded in the

Anthropological Theory of the Didactics (ATD, Chevallard, 2015), which is increasingly used in studies at tertiary level. Moreover, these studies introduce new methodological approaches: Question-Answer maps, essential for the epistemological analysis needed before designing an SRP, and also allowing a collective work of researchers in mathematics educations and university teachers (Florensa and Bosch[2]). Related to emerging theoretical approaches, we also would like to mention the increasing number of studies using discursive approaches. Such studies consider the teaching and learning phenomena as discursive phenomena, and use methods for analyzing discourses. In TSG-2, Nardi[14] raised the potential of the commognitive approach (Sfard, 2008) to observe discursive shifts in university mathematics, beyond the micro-level of a student working on a precise task. She claimed that this approach can inform a reform agenda. Besides the aforementioned frameworks, some studies proposed innovative conceptualization to address particular aspects of students' practices at tertiary level. Williams et al.[24] proposed a conceptualization of students' engagement in collective mathematical activity. Göller[4] proposed categories for the analysis of students' goals and strategies. Levin et al.[11] introduced a conceptualization of students' agency and autonomy in the context of mathematics teaching and learning at tertiary level. This conceptualization leads to consider that students' agency and autonomy can change in response to context and over time. We really look forward to seeing further advances to research in tertiary mathematics in the next ICME conference and in other events such as INDRUM, CERME, RUME or DELTA conferences.

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